

Bounds on masses of new gauge bosons in the 3 - 3 -1 models ¹

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Abstract

Contribution from new gauge bosons in the 3 - 3 - 1 models to the anomalous magnetic moment of the muon, mass difference of the kaon system and rare kaon decay are calculated and numerically estimated. Bounds on masses of new gauge bosons: bileptons and Z' are derived.

1 Introduction

The SuperKamiokande results [1] confirming non-zero neutrino mass call for the standard model (SM) extension. Among the known extensions, the models based on the $SU(3)_C \times SU(3)_L \times U(1)_N$ gauge group [2, 3] (hereafter 3 - 3 - 1 models) have the following intriguing features: firstly, the models are anomaly free only if the number of families N is a multiple of three. Further, from the condition of QCD asymptotic freedom, which means $N < 5$, it follows that N is equal to 3. The second characteristic is that the lagrangians of these models possess the Peccei-Quinn symmetry naturally, hence the strong CP problem can be solved in an elegant way [4]. The third interesting feature is that one of the quark families is treated differently from the other two. This could lead to a natural explanation of the unbalancing heavy top quarks in the fermion mass hierarchy. Recent analyses have indicated that signals of new particles in this model, bileptons [5] and exotic quarks [6] may be observed at the Tevatron and the Large Hadron Collider.

There are two main versions of the 3 - 3 - 1 models: the *minimal* model in which all lepton components $(\nu, l, l^c)_L$ of each family belong to one and same lepton triplet and a variant, in which right-handed (RH) neutrinos are included, i.e. $(\nu, l, \nu^c)_L$ (hereafter we call it the model with right-handed neutrinos [7, 8]). New gauge bosons in the minimal model are bileptons $(Y^\pm, X^{\pm\pm})$ carrying lepton number $L = \pm 2$ and Z' . In the second model, the bileptons with lepton number $L = \pm 2$ are singly-charged Y^\pm and *neutral* gauge bosons X^0, X^{*0} , and both are responsible for lepton-number violating interactions. Thus, with the present group extension there are five new gauge bosons and all these particles are heavy. Getting mass limits for these particles is one of the central tasks of further

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studies.

In this report, we summarize constraints on new gauge boson masses using various experimental data, namely, the AMMM [9], mass difference of the kaon system [10] and rare kaon decay [11].

2 Bounds on the bilepton masses from the AMMM

The anomalous magnetic moments of the muon (AMMM) is one of the most popular values in pursuing this aim. Despite not competitive with the anomalous magnetic moment of the electron (AMME) in precision, the AMMM is much more sensitive to loop effects as well as “New Physics” due to contributions $\sim m_\mu^2$, i.e. $\sim (200)^2$ enhancement in the AMMM relative to the AMME. Therefore the AMMM is a subject of both theoretical and experimental investigations.

2.1 The AMMM in the minimal model

Before we go into the detailed calculation, let us recapitulate some basic elements of the model (for more details see [12]). Three lepton components of each family are in one triplet:

$$f_L^a = (\nu^a, l^a, (l^c)^a)^T_L \sim (1, 3, 0), \quad (1)$$

where $a = 1, 2, 3$ is the family index. The charged bileptons with lepton number $L = \pm 2$ are identified as follows: $\sqrt{2} Y_\mu^- = W_\mu^4 - iW_\mu^5$, $\sqrt{2} X_\mu^{--} = W_\mu^6 - iW_\mu^7$, and their couplings to leptons are given by [13]

$$\mathcal{L}_l^{CC} = -\frac{g}{2\sqrt{2}} \left[\bar{\nu} \gamma^\mu (1 - \gamma_5) C \bar{l}^T Y_\mu^- - \bar{l} \gamma^\mu \gamma_5 C \bar{l}^T X_\mu^{--} + \text{h.c.} \right]. \quad (2)$$

It is to be noted that the vector currents coupled to X^{--} , X^{++} vanish due to Fermi statistics. To get physical neutral gauge bosons one has to diagonalize their mass mixing matrix. That can be done in two steps: At the first, the photon field A_μ and Z, Z' are given by [12]

$$\begin{aligned} A_\mu &= s_W W_\mu^3 + c_W \left(\sqrt{3} t_W W_\mu^8 + \sqrt{1 - 3 t_W^2} B_\mu \right), \\ Z_\mu &= c_W W_\mu^3 - s_W \left(\sqrt{3} t_W W_\mu^8 + \sqrt{1 - 3 t_W^2} B_\mu \right), \\ Z'_\mu &= \sqrt{3} t_W B_\mu - \sqrt{1 - 3 t_W^2} W_\mu^8. \end{aligned} \quad (3)$$

In the second step, we get the physical neutral gauge bosons Z^1 and Z^2 which are mixtures of Z and Z' :

$$\begin{aligned} Z^1 &= Z \cos \phi - Z' \sin \phi, \\ Z^2 &= Z \sin \phi + Z' \cos \phi. \end{aligned} \quad (4)$$

The mixing angle ϕ is constrained to be very small, therefore the Z and the Z' can be safely considered as the physical particles.

Now we calculate contributions from the bileptons and the Z' to the AMMM. It is known that heavy Higgs boson contribution to the AMMM is negligible [14], therefore the relevant diagrams are depicted in Fig.1. The first three diagrams come from the bileptons and their contributions are found to be

$$\delta a_\mu^B = \frac{g^2 m_\mu^2}{24\pi^2} \left(\frac{16}{M_X^2} + \frac{5}{4M_Y^2} \right), \quad (5)$$

where M_X , M_Y , m_μ stand for masses of the doubly-, singly-charged bileptons and of the muon, respectively. In the limit $m_\mu \ll M_{Z'}$ where $M_{Z'}$ is the Z' mass, the Z' contribution has the form [15]

$$\delta a_\mu^{Z'} = \frac{m_\mu^2}{12\pi^2 M_{Z'}^2} (g_V'^2 - 5g_A'^2). \quad (6)$$

Following [9] (using Eq. (6) therein) we get coupling of the muon to the Z'

$$g_V'(\mu) = \frac{g}{c_W} \frac{3\sqrt{1-4s_W^2}}{2\sqrt{3}}, \quad g_A'(\mu) = \frac{g}{c_W} \frac{\sqrt{1-4s_W^2}}{2\sqrt{3}}. \quad (7)$$

Substituting (7) into (6) we obtain the Z' contribution

$$\delta a_\mu^{Z'} = \frac{g^2}{3c_W^2} \frac{m_\mu^2}{12\pi^2 M_{Z'}^2} (1 - 4s_W^2). \quad (8)$$

Therefore the total contribution from new gauge bosons in the minimal version to the AMMM becomes [9]

$$\delta a_\mu^{tm} = \frac{G_F m_W^2 m_\mu^2}{3\sqrt{2}\pi^2} \left[\frac{16}{M_X^2} + \frac{5}{4M_Y^2} + \frac{2(1-4s_W^2)}{3c_W^2 M_{Z'}^2} \right], \quad (9)$$

where $G_F/\sqrt{2} = g^2/(8m_W^2)$ is used.

Note that the Z' gives a positive contribution to the AMMM, while the Z gives a negative one as it is well-known in the SM. From Eq. (9) it follows that the bilepton contributions are dominant. By the spontaneous symmetry breaking (SSB) it follows that [9] $|M_X^2 - M_Y^2| \leq 3m_W^2$. Therefore it is acceptable to put $M_X \sim M_Y$ as it was done in [16]. In this approximation, Eq. (5) agrees with the original result in [16], and Eq. (9) becomes

$$\delta a_\mu^{tm} = \frac{G_F m_W^2 m_\mu^2}{\sqrt{2}\pi^2} \left[\frac{23}{4M_Y^2} + \frac{2(1-4s_W^2)}{9c_W^2 M_{Z'}^2} \right]. \quad (10)$$

A lower limit $M_Y \sim 230$ GeV at 95% CL can be extracted by the "wrong" muon decay $\mu \rightarrow e^- \nu_e \bar{\nu}_\mu$. Combining with the SSB, it follows [12] $M_{Z'} \geq 1.3$ TeV. With the quoted numbers ($M_X = 180, M_Y = 230, M_{Z'} = 1300$ GeV), the contributions to δa_μ^{tm} from

the bileptons and the Z' are 1.04×10^{-8} and 7.76×10^{-13} , respectively. The bilepton contribution is in a range of "New Physics" one [17] $\sim \mathcal{O}(10^{-8})$.

Putting a bound on "New Physics" contribution to the AMMM [18]

$$\delta a_\mu^{New \text{ Physics}} = (7 \pm 8.6) \times 10^{-9}, \quad (11)$$

into the l.h.s of (10) we can obtain a bound on M_Y . In [9] (see Fig. 2) we plot δa_μ^{tm} as a function of M_Y . For certainty we used $M_{Z'} = 1.3$ TeV quoted above. The horizontal lines are the upper and the lower limit from $\delta a_\mu^{New \text{ Physics}}$. From the figure we get a lower mass limit on M_Y to be 167 GeV. We recall that this limit is in a range of those obtained from LEP data analysis ($M_Y \geq 120$ GeV) (see Ref. 21 of [9]). In the near future, the E-821 Collaboration at Brookhaven would reduce the experimental error on the AMMM to a few $\times 10^{-10}$. In Fig. 3 of [9] we see that δa_μ^{tm} cuts horizontal line I ($\sim 4 \times 10^{-10}$) and line II ($\sim 1 \times 10^{-10}$) at $M_Y \approx 935$ GeV and $M_Y \approx 1870$ GeV, respectively. These lower bounds are much higher than those from the muon experiments.

2.2 The AMMM in the model with RH neutrinos

In this subsection we will calculate the AMMM in the model with RH neutrinos. Let us recapitulate some basic elements of the model (for more details see [7]). In this version the third member of the lepton triplet is a RH neutrino instead of the antilepton l_L^c

$$f_L^a = (\nu^a, l^a, (\nu^c)^a)_L^T \sim (1, 3, -1/3), l_R^a \sim (1, 1, -1). \quad (12)$$

The complex new gauge bosons $\sqrt{2} Y_\mu^- = W_\mu^6 - iW_\mu^7$, $\sqrt{2} X_\mu^0 = W_\mu^4 - iW_\mu^5$ are responsible for lepton-number violating interactions. Instead of the doubly-charged bileptons $X^{\pm\pm}$, here we have neutral ones X^0, X^{0*} . The SSB gives the bilepton mass splitting [19]

$$|M_Y^2 - M_X^2| \leq m_W^2.$$

As before one diagonalizes the mass mixing matrix of the neutral gauge bosons by two steps, and the last one is the same for both versions. At the first step we have

$$\begin{aligned} A_\mu &= s_W W_\mu^3 + c_W \left(-\frac{t_W}{\sqrt{3}} W_\mu^8 + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right), \\ Z_\mu &= c_W W_\mu^3 - s_W \left(-\frac{t_W}{\sqrt{3}} W_\mu^8 + \sqrt{1 - \frac{t_W^2}{3}} B_\mu \right), \\]Z'_\mu &= \sqrt{1 - \frac{t_W^2}{3}} W_\mu^8 + \frac{t_W}{\sqrt{3}} B_\mu. \end{aligned} \quad (13)$$

Due to smallness of mixing angle ϕ we can consider the Z and the Z' as the physical particles. Due to its neutrality, the bilepton X^0 does not give a contribution and in this

case, the relevant diagrams are only two last (c) and (d). The contribution from the singly-charged bilepton and the Z' in Fig. 1(c) and 1(d) is

$$\delta a_\mu^{tr} = \frac{G_F m_W^2 m_\mu^2}{12\sqrt{2}\pi^2} \left\{ \frac{5}{M_Y^2} - \frac{[5 - (1 - 4s_W^2)^2]}{2c_W^2(3 - 4s_W^2)M_{Z'}^2} \right\}. \quad (14)$$

In the considered version the Z' gives a negative contribution. However, the total value in r.h.s of Eq. (14) is positive (an opposite sign happens when $M_{Z'} \leq 0.3 M_Y$ which is excluded by the SSB).

Putting the Z' lower mass bound to be 1.3 TeV [12] and $M_Y = 230$ GeV we get the bilepton and the Z' contributions to δa_μ^{tr} , respectively: 4.75×10^{-10} and -7.87×10^{-12} . This implies that the contribution of the new gauge bosons in the considered version is in two order smaller than an allowed difference between theoretical calculation in the SM and present experimental precision. However, putting two previous values for δa_μ^{tr} we get lower bounds on the bilepton masses to be about 250 GeV (I) and 500 GeV (II) (see Fig. 4 in [9]).

3 Constraint on Z' mass from the kaon mass difference Δm_K

In the SM, flavour changing neutral current (FCNC) is completely suppressed by GIM mechanism at tree level. In the second or higher orders, this suppression is not complete due to quark mass disparity [20]. In the $3 - 3 - 1$ model we can have FCNC even at tree level. In the left-handed sector, since the third family has a different N charge from the first and second family, their gauge couplings to Z' are different, leading to FCNC through the mismatch between weak and mass eigenstates. Let us diagonalize mass matrices by three biunitary transformations

$$\begin{aligned} U'_L &= V_L^U U_L, \quad U'_R = V_R^U U_R, \\ D'_L &= V_L^D D_L, \quad D'_R = V_R^D D_R, \end{aligned} \quad (15)$$

where $U \equiv (u, c, t)^T$ and $D \equiv (d, s, b)^T$. The usual Cabibbo-Kobayashi-Maskawa matrix is given by

$$V_{CKM} = V_L^{U+} V_L^D. \quad (16)$$

Using the unitarity of the V^D and V^U matrices, we get FCNC interactions in down sector

$$\mathcal{L}_{ds}^{NC} = \frac{g_{CW}}{2\sqrt{3 - 4s_W^2}} \left[V_{Lid}^{D*} V_{Lis}^D \right] \bar{d}_L \gamma^\mu s_L Z'_\mu, \quad (17)$$

where $i = (d, s, b)$; in our case $i = b$.

From the flavour-changing neutral current interaction (17), we have the effective lagrangian

$$\mathcal{L}_{eff}^{\Delta S=2} = 3D \frac{\sqrt{2} G_F c_W^4}{3 - 4s_W^2} \frac{m_Z^2}{m_Z'^2} \left[V_{Lbd}^{D*} V_{Lbs}^D \right]^2 |\bar{d}_L \gamma^\mu s_L|^2. \quad (18)$$

From the effective lagrangian, it is straightforward to get the mass difference [10]

$$\Delta m_K = \frac{4G_F c_W^4}{3\sqrt{2}(3-4s_W^2)} \frac{m_Z^2}{m_Z'^2} [V_{Lbd}^{D*} V_{Lbs}^D]^2 f_K^2 B_K m_K. \quad (19)$$

It is expected that the Z' contribution to Δm_K is no larger than observed values [21]. Using the experimental values [22]

$$\Delta m_K = 3.489^{+0.009}_{-0.009} \times 10^{-12} \text{ MeV}, \quad (20)$$

$$m_K = 498 \text{ MeV}, \quad (21)$$

$$\sqrt{B_K} f_K = 3D \quad 135 \pm 19 \text{ MeV}, \quad (22)$$

we have

$$m_{Z'} \geq 2.63 \times 10^5 \eta_{Z'} [Re |V_{Lbd}^{D*} V_{Lbs}^D|^2]^{1/2} \text{ GeV}, \quad (23)$$

where $\eta_{Z'} = 0.55$ is the leading order QCD correction. From the present experimental data we cannot impose constraints on V_{Lbd}^{D*} and V_{Lbs}^D . Using the Fritzsch [23] scheme

$$V_{ij}^D \approx \left(\frac{m_i}{m_j} \right)^{1/2}, \quad i < j, \quad (24)$$

where i, j are family indices, we get the bound on Z' mass

$$m_{Z'} \geq 1.02 \text{ TeV}. \quad (25)$$

4 Constraint on Z' mass from the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

In the SM the decay is loop-induced semileptonic FCNC determined only by Z^0 -penguin and box diagram. It is worthwhile to mention that the photon-penguin contribution is absent in the decay since photon does not couple to neutrinos. We now move to discuss the semileptonic rare FCNC transition $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in the framework of 3 - 3 - 1 model and show how this decay can be used to get constraint on Z' mass. The Feynman diagram contributing to the considered decay is depicted in Fig. 2. In the 3 - 3 - 1 model, due to the FCNC interaction in (17) the decay can occur at tree level as in Fig. 2 of [11]. The decay amplitude is given

$$\begin{aligned} \mathcal{M}_{\pi^+ \nu \bar{\nu}} &= \frac{G_F}{2\sqrt{2}} \frac{m_W^2}{m_{Z'}^2} V_{Lbd}^{D*} V_{Lbs}^D \langle \pi^+(p_2) | \bar{s}_L \gamma_\mu d_L | K^+(p_1) \rangle \\ &\times \bar{\nu}(k_1) \gamma^\mu (1 - \gamma_5) \nu(k_2), \end{aligned} \quad (26)$$

where we have neglected Z' momentum compared with its mass. On the other hand, in the SM the tree-level amplitude for the semileptonic decay $K^+(p_1) \rightarrow \pi^0(p_2) e^+(k_1) \nu(k_2)$

is given

$$\begin{aligned}\mathcal{M}_{\pi^0 e^+ \nu} &= \frac{G_F}{2\sqrt{2}} V_{us}^* \langle \pi^0(p_2) | \bar{s}_L \gamma_\mu u_L | K^+(p_1) \rangle \\ &\times \bar{\nu}_e(k_1) \gamma^\mu (1 - \gamma_5) e(k_2).\end{aligned}\quad (27)$$

Isospin symmetry relates hadronic matrix elements in (26) and (27) to a very good precision [25]

$$\langle \pi^+(p_2) | \bar{s}_L \gamma_\mu d_L | K^+(p_1) \rangle = \sqrt{2} \langle \pi^0(p_2) | \bar{s}_L \gamma_\mu u_L | K^+(p_1) \rangle. \quad (28)$$

Neglecting differences in the phase space of the two decays, due to $m_{\pi^+} \neq m_{\pi^0}$ and $m_e \neq 0$, we obtain after summation over three neutrino flavours [11]

$$\frac{Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{Br(K^+ \rightarrow \pi^0 e^+ \nu)} = 6 \left(\frac{m_W^2}{m_{Z'}^2} \right)^2 \frac{|V_{Lbd}^{*D} V_{Lbs}^D|^2}{|V_{us}^*|^2}. \quad (29)$$

Using the experimental data [22]

$$\begin{aligned}m_W &= 80.41 \text{ GeV}, \quad |V_{us}| = 0.2196, \quad m_d = 7 \text{ MeV}, \\ m_s &= 115 \text{ MeV}, \quad m_b = 4.3 \text{ GeV}, \\ Br(K^+ \rightarrow \pi^0 e^+ \nu) &= 4.42 \times 10^{-2}\end{aligned}\quad (30)$$

and (24) we have

$$Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \frac{10923}{m_{Z'}^4 (\text{GeV})}. \quad (31)$$

We notice that the standard model result after including next-to-leading order QCD corrections for the decay is [24]

$$0.79 \times 10^{-10} \leq Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \leq 0.92 \times 10^{-10}, \quad (32)$$

while the present experimental values at Brookhaven [26]

$$Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 1.5_{-1.2}^{+3.4} \times 10^{-10}. \quad (33)$$

Therefore, if $3-3-1$ symmetry is realized in nature, we can expect that Z' contribution to the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is of order 10^{-10} . Putting the central value in (33) into (31), we get $m_{Z'} \simeq 2.3 \text{ TeV}$.

Now let us consider the decay in the minimal model. In this model the FCNC interaction is described by [12, 27]

$$\mathcal{L}_{ds}^{NC} = \frac{g_{CW}}{2\sqrt{3}(1-4s_W^2)} [V_{Lbd}^{D*} V_{Lbs}^D] \bar{d}_L \gamma^\mu s_L Z'_\mu. \quad (34)$$

Following the same steps as we have done in the r.h.n. model we obtain

$$\frac{Br_m(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{Br(K^+ \rightarrow \pi^0 e^+ \nu)} = \frac{2}{3} \left(\frac{m_W^2}{m_{Z'}^2} \right)^2 \frac{|V_{Lbd}^{*D} V_{Lbs}^D|^2}{|V_{us}^*|^2}. \quad (35)$$

The index m indicates that the branching ratio is calculated in the minimal model. Using (30) we find

$$Br_m(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \frac{1212}{m_{Z'}^4 (\text{GeV})}. \quad (36)$$

Using the measured decay branching ratio in (33) we get $m_{Z'} \simeq 1.7$ TeV. This result is consistent with constraints in [12] which come from muon decay and neutrino-nucleus scattering. It is worthwhile mentioning that the branching ratio is not sensitive to the value of $\sin^2 \theta_W$, while the expression of Δm_K is very sensitive to $\sin^2 \theta_W$.

In conclusion we emphasize that the new gauge bosons in the 3 - 3 - 1 models have lower mass limit in the range of TeV scale, and these models can be checked at the near future experiments.

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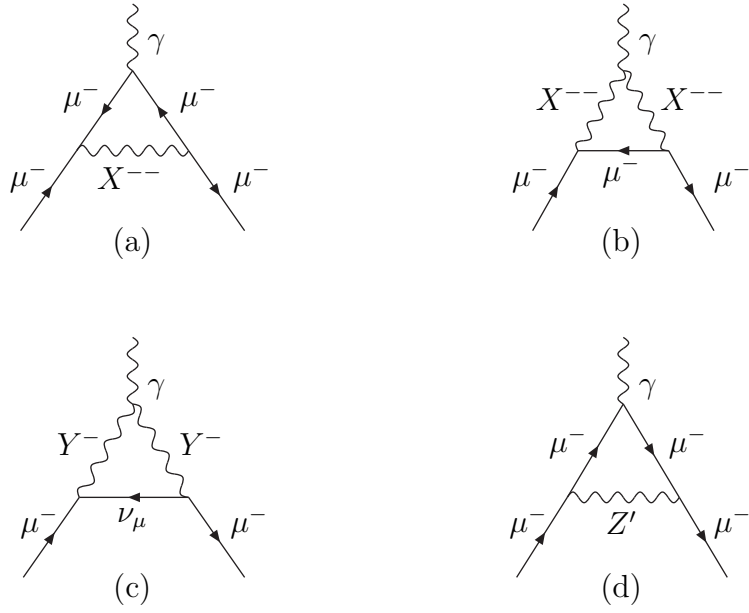


Figure 1: Diagrams contributing to the $(g_\mu - 2)/2$.

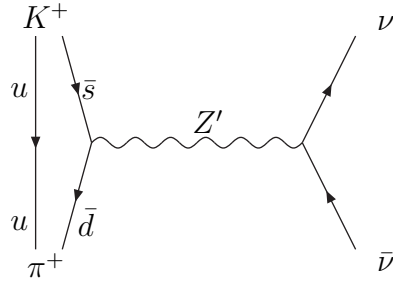


Figure 2: Feynman diagram for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in the 3 - 3 - 1 models.